ME5113 HW2 Report

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Q1a). Study the behaviors of Chebychev polynomial Tn(x)

The Chebyshev polynomials are two sequences of polynomials related to the cosine and sine functions, notated as Tn(x) and Un(x). They can be defined several ways that have the same end result; in this article the polynomials are defined by starting with trigonometric functions:

The Chebyshev polynomials of the first kind Tn(x) are given by:



And the Chebyshev polynomials of the second kind Un(x) are given by:



These polynomials satisfy the recurrence relations:

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Another important property of Tn(x) is orthogonality, i.e.

Diagram

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So, Chebyshev polynomials form an orthogonal set on the interval -1 <= x <= 1 with the weighting function (1-x^2)^(-1/2).

Also, Chebyshev polynomials has extremal property that Tn(x) are polynomials with the largest possible leading coefficient, whose absolute value on the interval [-1,1] is bounded by 1.

Chebyshev polynomials are important in approximation theory because the roots of Tn(x), which are also called Chebyshev nodes, are used as matching-points for optimizing polynomial interpolation. The resulting interpolation polynomial minimizes the problem of Runge's phenomenon, and provides an approximation that is close to the best polynomial approximation to a continuous function under the maximum norm, also called the "minimax" criterion. This approximation leads directly to the method of Clenshaw–Curtis quadrature.

Reference: <https://journalofinequalitiesandapplications.springeropen.com/articles/10.1186/1029-242X-2012-167>

<https://en.wikipedia.org/wiki/Chebyshev_polynomials>

Q1b). Study the behaviors of Legendre polynomial.

The Legendre polynomial Ln(z) is defined by:

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This is the contour integral where the contour encloses the origin and is traversed in a counterclockwise direction.

Also, The Legendre polynomials can also be generated using Gram-Schmidt orthonormalization in the open interval (-1,1) with the weighting function 1.

It is also orthogonal over (-1,1) with weighting function 1 and satisfy

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